Modelling Spatially Unrestricted Pedestrian Traffic on Footbridges

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Abstract

The research into modelling walking-induced dynamic loading and its effects on footbridge structures and people using them has been intensified in the last decade after some high profile vibration serviceability failures. In particular, the crowd induced loading, characterised by spatially restricted movement of pedestrians, has kept attracting attention of researchers. However, it is the normal spatially unrestricted pedestrian traffic, and its vertical dynamic loading component, that are most relevant for vibration serviceability checks for most footbridges. Despite the existence of numerous design procedures concerned with this loading, the current confidence in its modelling is low due to lack of verification of the models on as-built structures. This is the motivation behind reviewing the existing design procedures for modelling normal pedestrian traffic in this paper and evaluating their performance against the experimental data acquired on two as-built footbridges. Additionally, the use of Monte Carlo simulations is also investigated. Possible factors that cause discrepancies between measured and calculated vibration responses, including possibility of existence of pedestrian-structure dynamic interaction, are discussed.

Keywords: footbridge, vibration serviceability, spatially unrestricted pedestrian traffic, Monte Carlo simulations, human-structure interaction.

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Introduction

Since the swaying problem of the Millennium Bridge in London, that occurred in June 2000, a lot of research has been devoted to modelling pedestrian-induced dynamic loading and predicting the corresponding vibration response of footbridges (Dallard et al., 2001; Brownjohn et al., 2004a,b; Roberts, 2005; Kasperski, 2006; Živanović, 2006; Ricciardelli and Pizzimenti, 2007; Venuti et al., 2007; Macdonald, 2009; Ingólfsson et al. 2008). This contributed to educating the civil engineering community with respect to the effects that walking-induced dynamic force could have on vibration serviceability of a structure. Differently from a decade ago, most civil engineers dealing with footbridge design nowadays are aware of potential serviceability problems and are eager for more guidance in design.

Each footbridge resides in a different environment that defines possible loading scenarios to which the bridge could be exposed during its lifetime. Typically, these scenarios could be divided into: (1) single person loading, (2) normal spatially unrestricted traffic (i.e. traffic within which each individual could walk freely), (3) crowd loading (i.e. the walking of an individual is spatially restricted due to close proximity of other pedestrians), (4) group loading, and (5) vandal loading (usually involving jumping and/or bouncing).

These loading scenarios are more or less successfully defined in literature. However, there is no definite consensus on the estimation of the vibration response to the normal traffic although this scenario is most likely for most footbridges. This is in contrast with the fact that there are plenty of design procedures that define this loading scenario, but their reliability and limitations are not fully understood and therefore require further investigation. The “normal traffic” is also called “multi-person traffic” and “spatially unrestricted traffic” in this study, and refers to maximum density of around 0.3 people/m^2 (Venuti et al., 2007).

This paper critically reviews the existing procedures for modelling the spatially unrestricted pedestrian traffic and evaluates them against the experimental data collected on two as-built footbridge structures. In addition, a probabilistic procedure based on Monte Carlo simulations is employed for getting more detailed insight into structural performance. Possible interaction between walking people and a perceptibly moving structure is also investigated. Since both bridges are much more responsive in the vertical direction, only the vibration and the force model for this direction are studied.

The paper starts with an overview of some of the design methods currently available for estimating the vertical vibration of footbridges due to normal traffic. Then the current limited knowledge about the human-structure dynamic interaction in the vertical direction while walking is briefly reviewed. After this experimental investigation on two footbridge structures as well as their experimentally acquired modal properties and response data are described. Finally, the reviewed design procedures are employed to predict the vibration response of the two footbridges. This process identified some discrepancies between predicted vibration levels and the measured ones. The potential causes for these differences were discussed, including neglecting dynamic interaction between pedestrians and the perceptibly vibrating structure. The outputs of the analysis presented should help a designer to make an informed decision about the procedures to be used for modelling normal pedestrian traffic.

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Existing Design Procedures

This section presents some of the design procedures available to bridge designers for estimation of vibration response to the multi-person traffic. The key assumptions for each model are stated shedding some light on their possible area of application as well as on their shortcomings.

Over the years, it has been tempting to calculate the vibration response to multi-person traffic as a multiple of the response to a single person excitation and a factor which is a function of the number of people crossing the bridge at the same time. This approach originated from the work by Matsumoto et al. (1978) and is still popular due to its simplicity. The four design guidelines based on this approach are presented first. They all belong to the time-domain class of models. Then three frequency-domain models using ideas from random vibration theory and earthquake and wind engineering are described. Seven procedures presented are all recent advances in the field published in the last five years. They are mainly applicable to calculating the response in a single vibration mode. The section concludes with a description of a model used as a basis for time-domain Monte Carlo simulations, that could be used for statistical description of the structural response.

Eurocode 5

Eurocode 5, accepted in the UK in 2004 (EN, 2004) is a guideline concerned with design of timber bridges. However, the response model defined is not timber-specific and therefore could be used for check on bridges made of any material.

The calculation procedure assumes that the bridge is a simply supported beam-like structure. The amplitude of the acceleration response under the load from N pedestrians (present on the bridge at the same time) is given as:

$$a_N = 0.23 a_1 N k \tag{1}$$

where a_1 is the amplitude of the steady-state response from a single pedestrian (modelled as a stationary harmonic force matching the natural frequency of the bridge), given as:

$$a_1 = \begin{cases} \dfrac{200}{M\zeta} & \text{for } f_n \leq 2.5\,\text{Hz} \\ \dfrac{100}{M\zeta} & \text{for } 2.5\,\text{Hz} < f_n \leq 5.0\,\text{Hz}. \end{cases} \tag{2}$$

M, ζ and f_n are the total mass of the bridge span, the damping ratio of the relevant vibration mode excitable by walking and the natural frequency, respectively. A constant pedestrian force amplitude of 200 N is used for modelling the first harmonic, while 100 N is used for the second one. These forces are obtained assuming dynamic force induced by a single pedestrian (weighing 700 N) equal to 40% (280 N) and 20% (140 N) of their weight, for the first and second harmonic respectively (Butz, 2008b). These values are further reduced by factor 0.7 to account for the fact that the steady-state cannot be echieved due to limited time a pedestrian spends on the bridge (Butz, 2008b). Factor k in Equation 1 reduces the number of synchronised people in the crowd if the bridge has the natural frequency away from the average pacing rate of pedestrian population (Figure 1a).

Limitations of the procedure are: (1) it is applicable to beam-like structures only, (2) it does not take into account that the force amplitude is a function of the walking frequency, and (3) the multiplication factor is linearly dependent on crowd size N.

ISO 10137

In the ISO 10137 standard (ISO, 2007) dealing with serviceability of walkways against vibrations, a load model for the vertical force induced by one pedestrian is given as:

$$F_1(t) = W\left(1 + \sum_{n=1}^{k} \alpha_n \sin\left(2\pi n f_p t + \phi_n\right)\right) \tag{3}$$

where W is the weight of the pedestrian, α_n is the dynamic loading factor (DLF) i.e. the proportion of the walking force harmonic with respect to W, ϕ_n is the phase angle for the nth load harmonic while f_p is the pacing frequency (normally in the range 1.2-2.4 Hz). The DLF for the first harmonic is defined as a function of the walking frequency f_p, i.e. $\alpha_1 = 0.37(f_p - 1)$, while the higher harmonics are defined as independent from the pacing rate ($\alpha_2 = 0.1$, $\alpha_3 = \alpha_4 = \alpha_5 = 0.06$). For a group of N uncoordinated pedestrians, the dynamic load in Equation 3 is multiplied by $\sqrt{N}$ to obtain the total effective pedestrian load. The guidance does not specify, however, the step frequency to be used in Equation 3, when calculating the response to single person loading. Presumably, it was meant to be the frequency that could match, by one or more forcing harmonics, the resonant frequency of the vibration mode(s) of interest. Also the guide does not explicitly specify if the force model is stationary or moving across the bridge, although the latter option seems more probable based on definitions in Appendix C.1.2. (ISO, 2007).

Therefore, differently from Eurocode 5, ISO 10137 defines the force, rather than the response, model. This makes the response calculation more robust than in Eurocode 5, since the modal properties to be used in calculations should be estimated for the actual structural layout rather than assumed to correspond to a beam-like structure. However, the fact that the loading model for multi-person traffic is $\sqrt{N}$ multiple of the response to a single pedestrian often yields considerable overestimation of the actual vibration response due to overconservative assumption in the model that all people walk with the same pacing frequency and randomly distributed phase (Ingólfsson et al. 2007).

French Sétra guideline

In the design guidelines presented by the French road authorities (Sétra, 2006) two different load models are developed; one for sparse and dense crowds, and the one for very dense crowds. The former model (characterised by pedestrian density of about 0.5 to 0.8 pedestrians/m^2) is more relevant for the normal traffic studied in this paper. The model is based on the assumption that the probability distribution of the pacing rate within traffic follows Gaussian distribution. The dynamic load per unit area is defined as:

$$f_N(t) = 10.8\frac{F_0}{A}\sqrt{N\zeta}\psi\cos\left(2\pi f_n t\right), \tag{4}$$

where A is the area of the bridge deck, N is the number of people in the crowd, F_0 is the load amplitude of a single pedestrian (280 N for the first and 70 N for the second harmonic), f_n is the natural frequency, ζ is the damping ratio and $\psi \in [0, 1]$ is a factor that reduces the load for frequencies away from the average pacing rate (Figure 1a). For each vibration mode with frequency below 5 Hz, a load case should be created according to Equation 4 and applied to the deck at the corresponding modal frequency. The direction of the load should be defined to correspond to the sign of the mode shape, to simulate the worst case scenario. Finally, the steady-state response to the load applied is considered as the estimate of the peak vibration response under crowd of people, with 95% probability of non-exceedance. In developing this model 500 simulations, each lasting long enough to cross the bridge twice, were executed for each configuration of interest, and a peak response with 5% probability of exceedance was used to fit the force model. The contribution of pedestrians towards the increase of the modal mass of the bridge is taken into account by increasing the modal mass in the dynamic model. The effect of pedestrians on the structural damping is, however, not accounted for.

As most other guidelines, the load model includes one harmonic at a time only, i.e. the multi-harmonic excitation of two or more modes is not considered. This is, however, not of concern except for rare bridges that have vibration modes with close to integer ratio between the natural frequencies.

UK NA to Eurocode 1

UK National Annex to Eurocode 1 (BSI, 2008) defines models for groups of people either walking or jogging and for 'crowd' loading. Similarly to Sétra, the crowd model refers to traffic that is denser (0.4 pedestrians/m^2 or above) than the spatially unrestricted traffic considered in this paper. However, this is the only model in the code that deals with a stream of pedestrians and is used here as closest represent of multi-person traffic under study. The load is defined as a load per unit area, with the load sign matching that of the mode shape:

$$f_N(t) = 1.8\frac{F_0}{A}k\sqrt{\frac{\gamma N}{\lambda}}\sin(2\pi f_n t), \tag{5}$$

where A is the area of the deck, F_0 is the reference loading of 280 N and f_n is the natural frequency of the relevant vibration mode. Factor γ takes into account the lack of correlation between people in the crowd. Interestingly, for a continuous stream of people it is a linear function of the damping ratio ζ and is equal approximately to $\gamma = 7.40\zeta$ (based on graph NA2.44.5. in the code). Factor k accounts for the excitation potential of the relevant forcing harmonic and probability of walking at the given resonant frequency (Figure 1a), and λ is the factor that is used to adjust the number of effective pedestrians depending on their position with regard to mode shape ordinates. It is defined as:

$$\lambda = \frac{\int_0^L |\phi(x)|/\phi_{max}\mathrm{d}x}{L}, \tag{6}$$

where L is the length of the bridge (or the loaded area), and $\phi(x)$ and ϕ_{max} are the mode shape along the bridge and its maximum ordinate, respectively. For a sinusoidal mode shape $\lambda = 0.64$, and the dynamic load in Equation 5 can be written in the same format to that of Sétra in Equation 4, with the multiplying constant equal to approximately 6.1 instead of 10.8, and k instead of ψ. This shows that the magnitude of the load model suggested by UK NA is about 56% of the magnitude in the Sétra load model (for sine mode shape and $k = \psi$). The difference is originating in different choice of the percentile value in the two models. Namely, according to Barker and Mackenzie (2008), whose numerical studies were used as a basis for UK NA, Sétra uses four standard deviations of the response as an estimate of the peak response, while UK NA choses 2.5 standard deviations for the same purpose. Less conservative estimate of UK NA is chosen with logical reasoning that some exceedance of the predicted response should be allowed in real life.

Like in Sétra, the UK NA force model has been calibrated against extensive Monte Carlo response simulations, such that the steady-state response calculated using the simplified method, equals a selected percentile peak acceleration obtained in the simulations.

Brownjohn et al. (2004b)

All previous models have been defined in the time-domain, and they assume that the effects of multi-person traffic could be simulated by multiplying the effect of a single person walking at a single (natural) frequency by an appropriate factor. Differently, the model defined by Brownjohn et al. (2004b) is defined in the frequency domain and it uses a Gaussian distribution of pacing rates as input parameter.

The power spectral density (PSD) of the force $S_{P,n}$ induced by N pedestrians is defined to approximately be:

$$S_{P,n} = \frac{N}{2n} W^2 \phi(f_p) G_n^2(f_p), \tag{7}$$

where n is the forcing harmonic considered, W is the average pedestrian weight, $\phi(f_p)$ is the probability distribution of pacing rate while $G_n(f_p)$ is the dynamic loading factor (which is dependent on pacing frequency for the first harmonic, i.e. $G_1(f_p) = 0.37f_p - 0.42$, but is constant for higher harmonics, i.e. $G_2 = 0.053$, $G_3 = 0.042$, $G_4 = 0.041$, $G_5 = 0.027$ and $G_6 = 0.018$).

Once the PSD of the loading is calculated, the PSD of the acceleration response $S_a(f)$ to the relevant harmonic of the walking force could be obtained using (Newland, 1993):

$$S_a(f) = |H(f)|^2 S_{P,n} \tag{8}$$

where $H(f)$ is the accelerance frequency response function of the bridge. The square root of the area under PSD function is then calculated to estimate the root-mean-square (RMS) of the vibration response.

The model is further developed, based on resemblance with the turbulent buffeting wind loading, to account for the effects of mode shape and the synchronisation between pedestrians. The synchronisation is accounted for via a coherence function $\mathrm{coh}(f, z_1, z_2)$. The final PSD of the acceleration response (for the fundamental harmonic) is then written as:

$$S_a(f) = \psi_z^2 |H(f)|^2 S_{P,1} \int_0^L \int_0^L \psi_{z1}\psi_{z2} \mathrm{coh}(f, z_1, z_2) \mathrm{d}z_1 \mathrm{d}z_2, \tag{9}$$

where ψ_z, ψ_{z1} and ψ_{z2} are the mode shape ordinates at the response point z, and at the two points z_1 and z_2 denoting positions of each pair of pedestrians, respectively. However, the coherence function featuring the equation is only defined for two extreme cases: no synchronisation and perfect synchronisation.

Butz (2006, 2008a)

Butz (2006) went a step further to account for not only probability distribution of step frequencies but also of pedestrian mass, forcing amplitude and pedestrian arrival times in a stream of pedestrians. Using Monte Carlo simulations she derived PSD functions for vibration response. Taking into account interdependence of some parameters (such as walking speed, step frequency and stream density), the 95th percentile of the peak modal acceleration amplitude was estimated via RMS acceleration value σ_a:

$$a_{peak,95\%} = k_p \sigma_{\mathrm{a}} \tag{10}$$

where k_p is an empirical peak factor that converts the RMS into 95th percentile peak acceleration. The factor is a function of crowd density and has values of 3.92, 3.80 and 3.74 for crowd densities around less than or equal to 0.5, 1.0 and 1.5 people/m^2, respectively. The RMS value is given as:

$$\sigma_{\mathrm{a}} = \sqrt{\frac{C \cdot k_{\mathrm{F}} \cdot N}{M_i^2} k_1(f_i) \cdot \zeta_i^{k_2(f_i)}} \tag{11}$$

where M_i, ζ_i and f_i are the modal mass, damping and frequency of mode i, respectively, N is the number of pedestrians on the bridge, C and k_{F} are empirical factors depending on the crowd density, while $k_1(f_i)$ and $k_2(f_i)$ are both polynomial functions determined from Monte Carlo response simulations (Butz, 2006). This spectral load model is applicable for simply supported structures/spans only, for modes in which the response in the first harmonic is most important and to the traffic within which the mean step frequency coincides with the natural frequency of the structure. Research Fund for Coal and Steel published a guideline for footbridge design (HIVOSS, 2008) that incorporates this model. Butz improved the model in 2008 by introducing a factor k_{red} that reduces the previously calculated acceleration to account for mismatch between the mean walking frequency and the natural frequency of the structure.

Response spectrum method

Inspired by earthquake engineering, a response spectrum methodology has been suggested as a simple way of dealing with vertical human induced vibrations (Georgakis & Ingólfsson, 2008; Ingólfsson et al., 2008). In essence, the peak modal acceleration $a_{\mathrm{R}}(T_R, r)$ of a single span simply supported reference bridge (span length 50 m, damping ratio 0.5%, modal mass 100000 kg) under two reference crowd populations was found via time-domain simulations involving stochastic treatment of arrival times and pedestrian characteristics. This was calculated as a function of ratio r between the relevant natural frequency f_n and the mean pacing rate in the pedestrian traffic and the return period T_R (i.e. mean time between two successive occurrences of a particular vibration level). The two reference populations are defined via the mean step frequencies equal to either 1.8 Hz or 2.0 Hz, respectively, with the standard deviation of 0.1 Hz, and pedestrian flow following the Poisson distribution with the mean arrival rate of one person/s. The peak modal acceleration (in mode n) of an actual bridge under an actual crowd of people could then be obtained by multiplying the reference acceleration by empirical multiplication factors:

$$a_n(T_R, r) = a_{\mathrm{R}}(T_R, r) \frac{M_0}{M_n} \sqrt{\lambda} \beta_\zeta \beta_L \beta_\Phi \tag{12}$$

where M_n and M_0 are the modal masses of the actual and the reference bridge, respectively, and λ is the actual pedestrian flow rate. The parameters β_ζ, β_L and β_Φ depend on the actual damping, bridge length and and mode shape, respectively. In addition, the first two parameters also depend on the ratio r (Ingólfsson et al. 2008). The reference peak acceleration was found using extreme value analysis of the simulated response to obtain:

$$a_{\mathrm{R}}(T_R) = A_\mu e^{\frac{-(D_\mu - r)^2}{B_\mu}} + C_1(T_R) e^{\frac{-\left(D_\psi - r\right)^2}{B_\psi}} + C_2(T_R). \tag{13}$$

Parameters A_μ, D_μ, B_μ, D_ψ and B_ψ are constants determined from the simulations and $C_1(T_R)$ and $C_2(T_R)$ are functions that incorporate the increase in acceleration response with increasing return period (Ingólfsson et al., 2008). This methodology is intuitive to follow since it accounts for influence of various parameters separately. However, the factorisation used implies that various parameters are mutually independent which might not necessarily be the case.

Monte Carlo simulations

Monte Carlo simulations of pedestrian traffic in this paper are based on a single person model that includes the frequency content of the walking force up to the fifth harmonic (Živanović et al., 2007). The magnitudes of each harmonic and subharmonic (originating in slightly different forces induced by left and right leg) in the force spectrum were modelled using a set of exponential functions that were found to be the best fit for the mean force spectra measured across different individuals. A typical force spectrum is shown in Figure 1b. When overlaying the force spectra for people walking at various pacing frequencies and generating different dynamic loading factors (DLFs) the x-axis in each spectrum was normalised by pacing frequency while the y-axis was normalised by the actual DLF. Resulting model therefore requires the information about the pacing rate and the DLF for each (sub)harmonic for a particular pedestrian to reconstruct the magnitude of the force spectrum. Assuming that the phases follow uniform distribution, the time domain force for the individual could be reconstructed and applied to each mode of interest to get modal responses.

This model was successfully verified on as-built structures for single person loading scenario (Živanović et al., 2007). The model can be used for modelling multi-person traffic by treating it as a traffic of multiple single persons, providing the distribution of the arrival time for pedestrians is known. Pacing frequency and DLFs are needed for reconstruction of the force time history. The pacing frequency should be chosen from the Gaussian distribution that best reflects the population crossing the bridge. The DLFs for five main harmonics should also be chosen from Gaussian distributions, with the mean value for the first harmonic dependent on the pacing frequency and standard deviation equal to 16% of the mean value (Kerr, 1998). The mean values for other four harmonics are equal to 7, 5, 5 and 3% of the weight, respectively. Subharmonics are, on the other hand, defined deterministically as linear functions of the DLF for the first harmonic (Živanović et al., 2007) due to current lack of statistically more reliable data. Having all DLFs and pacing frequencies defined, the time-domain force for each pedestrian can be calculated. Time that a pedestrian spends on the bridge depends on their speed, equal to the product of the pacing frequency and the step length. The latter parameter could be modelled as a Gaussian distribution (mean value of 0.75 m, standard deviation of 0.075 m). If arrival time for a person is taken from an appropriate Poisson distribution, the exact time for entering and leaving the bridge for each individual could be found. After inputting all parameters the Monte Carlo simulations that generate the time-domain forces induced by individual pedestrians could be conducted. After applying these forces to a bridge under consideration, the total response of the bridge could be found. The main advantage of this approach, apart from taking multi-harmonic nature of the walking force, is that simulations result in response time series that could then be statistically analysed. A disadvantage is that the simulations could be quite time consuming.

Human-Structure Interaction While Walking

When people walk on a structure vibrating perceptibly in the horizontal lateral direction, they might change their walking pattern and adapt to structural movement to preserve their body balance. A lot of research that investigates this feedback that people consciously or unconsciously receive from the structure and how it changes the force induced into the structure was conducted over the last decade (Dallard et al., 2001; McRobie et al., 2003; Roberts, 2005; Ricciardelli and Pizzimenti, 2007; Macdonald, 2009). Some studies concluded that people's effect on the structure could be modelled as negative damping, leading to excessive vibrations when a sufficient number of people occupy the bridge. However, when concerned with the human-structure interaction in the vertical direction less research is available. Some studies indicated that walking people add damping to the system (Willford, 2002; Brownjohn et al., 2004a; Živanović et al., 2009), similarly to the well-known effect that passive (standing) people have on the structure supporting them (Sachse, 2002). This suggests that human-structure interaction (HSI) is expected when people are walking across a perceptibly moving structure. Recently, HSI in the vertical direction has been studied by Georgakis and his co-workers (Georgakis, 2009) who found that the pedestrian force has a component which extracts energy from the structure which can be considered as an overall increase in the structural damping. However, none of the models reviewed in the previous section accounts for the HSI. This should be borne in mind when evaluating the viability of the models studied.

Full Scale Measurements on Two Footbridges

This section describes the footbridges investigated and summaries the experimental data collected. The two bridges are situated in the capital cities of Iceland and Montenegro.

Description of footbridges

The Reykjavik City Footbridge (RCF) is located in the Icelandic capital. It creates a pedestrian link, across an urban highway, between the University of Iceland and the city centre. In plan, the bridge has a spiral shape with a total length of 160 m, divided into eight spans, where the main span is 27.1 m (Figure 2a). It is built as a continuous post-tensioned concrete beam without any expansion joints, supported on guided elastomeric bearings at the end abutments and on cast-in, circular steel-concrete composite columns, with diameter of 500 mm, at the intermediate supports. At one end of the main span, spiralling steps diverge from the bridge deck to the ground to shorten the walking distance of the pedestrians. The cross section of the bridge deck is 3.2 m wide and has a variable thickness, from 170 mm at the edges, increasing to 700 mm at the centreline (Figure 2b). Four of the columns are supported on piles and the rest on a compacted fill (Gudmundsson et al., 2008).

The Podgorica Bridge (PB), considered to be lively in the vertical direction, is a steel box girder footbridge residing in the city centre of Podgorica. Its length is 104 m, with 78 m between inclined columns (Figure 3). Along its whole length the box girder is stiffened by longitudinal and transverse stiffeners, as shown in Figure 3. The connection between the inclined columns and box girder is enhanced by vertical stiffeners as well as by a concrete slab cast over the bottom steel flange forming a composite slab structure inside the box. The aim of this enhancement is to strengthen this part of the box section so that it can resist large column reaction forces and compression due to negative bending moments. A similar layout of the steel-concrete composite slab exists around the midspan of the footbridge, but the concrete is cast over the top flange of the box girder (Figure 3).

Measured modal properties

To identify the modal properties of the RCF a set of frequency response functions (FRFs) was acquired experimentally. The random excitation was generated using an electrodynamic shaker and measured by a piezoelectric accelerometer attached to the shaker armature. The force balanced accelerometers were used for measuring the responses in the vertical direction across measurement points shown in Figure 2a. Seven vibration modes having natural frequency below 5 Hz were identified (Živanović et al., 2010). First two modes (Figure 4a,b) could be excited by the first walking harmonic. Their modal masses are 575000 kg and 46000 kg, respectively. The large modal mass for the first mode is a consequence of engagement of columns, foundations and soil in vibration in this mode. As such it is difficult to have high confidence in experimental estimate of this modal mass especially because the corresponding FE model (that could theoretically be used for comparison with the measured data) would also suffer from uncertainties related to modelling soil-structure interaction (SSI). On the other hand the modal mass for mode 2 is more reliable and agrees well with estimates from the FE model (that neglects SSI).

Similarly, by employing an FRF-based modal testing on the PB, the first symmetric vertical (1V) vibration mode (Figure 4c) at 2.04 Hz having modal mass of 58000 kg and extremely low modal damping ratio of 0.26% was identified as the source of the liveliness of the bridge (Živanović et al., 2006). The low damping ratio is found to be independent from the vibration amplitude typically generated by pedestrian traffic. The measured modal properties are quite reliable estimates obtained from at least two different methods. The detailed description of the testing and parameter estimation procedures is described elsewhere (Živanović et al., 2006).

Monitoring tests

Since the RCF is rarely busy, the pedestrian traffic tests were conducted in a controlled manner with help of a group of 38 volunteers. They were asked to walk with their usual pacing rates over the three spans bounded by test points 5, 105, 17 and 117 (Figure 2a). They were introduced to the tested area from both directions at predefined times, generated from a Poisson distribution with mean arrival time of 12 people/minute. The test lasted around 16 minutes, during which time the vertical vibration response at test points 11, 111, 7 and 15, which are midspan points for the three spans (Figure 2a), were recorded. Differently, on the PB the vibration and normal pedestrian traffic were monitored using two video cameras and an accelerometer placed at the midspan point. The monitoring test lasted 45 minutes.

Results from monitoring tests

Properties of pedestrian traffic

Analysis of video records on RCF established that the mean pacing rate in the population of the volunteers was 1.95 Hz with standard deviation of 0.15 Hz. Considering the total pedestrian flow of 24 people/min and that in average about 50 s is required to cross the three spans (with an average speed of 1.4 m/s), it could be concluded that an average number of people present on the three spans at any time was around 20, with an average density being 0.1 pedestrians/m^2.

Similarly on PB it was found that the mean pacing rate was 1.87 Hz and standard deviation 0.18 Hz. It was also established that the arrival times from the two ends of the bridge follow Poisson distributions with the mean values of 6.3 and 5.9 people/minute. The average time required to cross the bridge was around 75 s, meaning that on average 15 people (0.05 pedestrians/m^2) were present on the bridge at any time.

Responsive vibration modes

The strongest responses on the RCF were recorded at TP11 and TP111, and they were almost the same. This was the reason to concentrate the further analysis on TP111 only. The power spectral density (PSD) of the measured acceleration response is shown in Figure 5a. The most energy in the response was concentrated in first two modes, which was the reason to concentrate further analysis on these modes only.

On the PB mode 1V at 2.04 Hz was practically the only contributor to the footbridge response, as shown in Figure 5b, due to its natural frequency being close to the usual pacing rate of people crossing it, and very low modal damping ratio. Note that due to presence of people the natural frequency of the system dropped from 2.04 Hz to 2.00 Hz. This was accounted for in all response estimates calculated in this paper. Assuming that the new human-structure system has the same modal stiffness as that of the empty bridge, the modal mass of the new system with natural frequency of 2 Hz is found to be 60350 kg. This updated modal mass value was also used in all simulations.

Response data

Measured response on the RCF (grey line in Figure 6a) was band-pass filtered (with cut-off frequencies at 1.50 and 2.15 Hz) to get the modal response in mode 1 (black line in Figure 6a). Since the two modes are quite close to each other and difficult to separate using conventional filters, the filtering was done by reconstructing the modal time domain signal using frequency lines from the frequency band of interest in the measured spectrum. In this way an ideal rectangular filter with sharp cut-off frequencies was implemented.

Due to the narrow band nature of the response, the maximum measured acceleration of the bridge is non-deterministic and will in general depend on the length of the observation time (i.e. the return period of an observed event). Therefore, a statistical treatment of the measured response would be a useful means of comparison with the outputs from the other models. For this reason the cumulative distributions of absolute value of both local peak (i.e. peak acceleration per cycle) and instantaneous modal acceleration are shown in Figure 6b. In addition, a cumulative distribution of peak values per 100 s data blocks (that correspond to two average crossing times) are presented. Same kind of data are presented for mode 2 (using cut off frequencies for band pass filtering at 2.15 and 2.70 Hz) of the RCF and mode 1V on PB in Figure 6c-f.

The RMS for measured acceleration on the RCF was 0.06 m/s^2. The contribution of the first mode was 0.03 m/s^2 (or about 25% in terms of energy of the response) and of the second mode 0.05 m/s^2 (or about 70% in terms of energy). On Podgorica bridge the RMS of the response in mode 1V was 0.13 m/s^2, which was more than 90% of the total response energy.

Vibration Response Estimates

This section presents the results of vibration estimation for the two footbridge structures using all methods described previously. The results are presented individually for each mode of interest given that most design procedures are defined for a single mode. This is a useful separation since human sensitivity to vibration varies with vibration frequency, and quoting response in each mode could be used (in practice) as a quick estimate of each mode contribution to human response to vibration. The results are evaluated against the benchmark measurement data.

Evaluation of design models

Most models to be evaluated require some degree of interpretation by the user. The way they are interpreted in this paper is explained on the example of the RCF. The same reasoning has then been followed for the PB.

Apart from modal properties of a structure analysed, some models (such as the Eurocode 5) require the mass of the span as an input parameter. This can be calculated knowing the mass per unit length: 2800 kg/m for the RCF and 2500 kg/m for the PB. Additionally, for those models applicable to beam-like structures the calculations are limited to the main spans of the two structures. These are equal to 27.1 m on the RCF and 78.0 m on the PB. Therefore, for these cases the number of people present on the main span only enters the calculations. This is obtained by scaling the total number of people (20 people on the RCF and 15 on the PB) down to the average number present on the main span only, i.e. 8 people for the RCF and 11 for the PB.

Reykjavik City Footbridge

The vibration estimates from all the models are listed in Table 1, with the corresponding measured values in square brackets as well as percentage error relative to the measured values. It can be seen in Table 1 that most procedures underestimate the response in mode 1. This is, at least partly, due to overestimation of the (experimentally determined) modal mass that is used in calculations. This is the reason to use modal response in mode

2 as a basis for evaluating performance of the models in this section.

Eurocode 5 procedure (EN, 2004) could only be used for estimating the response to traffic on the main span. The calculated response overestimates the measured peak response and is unlikely to ever be achieved on this bridge. Deviation of the mode from the assumed sinusoidal shape is partly responsible for the overestimation of the response. In addition k factor equal to 1 overestimates the pacing frequency in the actual crowd. The resonance pacing frequency assumed (2.33 Hz) is about 2.5 standard deviations away from the actual mean pacing rate of 1.95 Hz. Another source of the error is probably the multiplication factor value that is used for representing the effects of multi-person traffic. On the other hand the assumption of single pedestrian force of 280 N for walking at pacing rates matching resonant frequencies, being in this case above 2 Hz, is likely to be close to the actual single person loading.

For implementation of ISO 10137 (ISO, 2007) it was assumed that the load traverses the bridge at constant velocity. Weight W and step length, that are not defined in the code, were chosen as 750 N and 0.75 m, respectively. The modal force for each mode was calculated by modulating the force $F_1(t)$ (Equation 3) by the appropriate mode shape (Figure 4). The vibration response obtained was then multiplied by $\sqrt{N}$ to calculate the vibration response to multi-person traffic (Table 1). The response in mode 2 is overestimated by factor around 4. The force amplitude for a single person defined in the model is quite reasonable, and modal properties of the bridge are as measured. It remains that the main source of the error is the conservative multiplication factor defined as $\sqrt{N}$.

The response defined in Sétra (2006) guideline is calculated for the case of the actual average crowd density of about 0.1 people/m^2 (corresponding to 20 people occupying the three spans), instead of minimum of 0.5 people/m^2 defined in the guideline. The contribution of these people to the modal mass of the mode considered was taken into account by assuming the uniform distribution of pedestrians along the three spans, and then scaling their masses by the square of the mode shape ordinates. The modal response calculated was multiplied by the appropriate mode shape ordinate to get the vibration response at TP111 in each mode. The calculated 95th percentile of the peak response on the RCF is 0.34 m/s^2 in mode 2. Unfortunately, not enough measured data are available to evaluate these estimates reliably. Still the test data were post processed following the procedure used in the guideline: the measured response was divided in nine slices, each lasting 100 s. The 95th percentile of the extracted peaks was found to be 0.27 m/s^2 (Table 1), but only after linear interpolation between points with approximately 10% and 0% probability of exceedance, due to lack of data points (see Figure 6). The comparison for mode 2 suggests that Sétra slightly overestimates the response in this mode.

It would be expected, based on background information presented earlier, that UK NA model (BSI, 2008) produces almost two times lower estimate of the response compared with the response from Sétra. However, the estimates (Table 1) are only slightly lower than Sétra's. This is due to the difference in definition of factors k and ψ as well as due to amplitude deviation of the mode shape from the sinusoidal function that influences λ parameter in the UK NA model, with the former factor having stronger influence on the results. If the calculated values are compared with measured vibration level which is 2.5 standard deviations (equivalent to 2.5RMS value due to zero mean of the signal) away from the mean (given in square brackets in Table 1) then it follows that the response in mode 2 is overestimated. The calculated value for mode 2 was never reached during the test.

The model by Brownjohn et al. (2004b) was implemented under assumption that pedestrians are uncorrelated. The predicted RMS value of the response in mode 2 (0.06 m/s^2) is quite a good estimate of the measured response.

The 95th percentile peak vibration response according to Butz (2008a) guideline was calculated considering the main span only. The resulting response (Table 1) underestimates the measured value significantly. The model consists of several empirical factors which make it difficult to locate the main sources of discrepancies. It is interesting that the estimate of the response in mode 2 was overestimated (0.61 m/s^2) initially, i.e. before introducing the reduction factor that accounts for actual mean pacing rate in the crowd. It might be that the calibration of this factor requires improvement.

Unlike the other procedures, when determining the peak acceleration using the response spectrum method, the user should determine the return period for which the acceleration is evaluated. Here a time window of 100 s (corresponding to time needed to cross the three spans twice) has been selected. The peak acceleration with 50% probability of exceedance $a_{50\%}$ (i.e. return period of 3.3 minutes) was determined (Table 1). For mode 2 the estimated peak (0.09 m/s^2) is lower than that measured (0.15 m/s^2). The peak acceleration with 5% probability of exceedance during 100 s (i.e. return period of 33 minutes) is also shown in the table. In both cases, the response spectrum underestimates the response and most probable explanation for this difference is the fact that the standard deviation of the pacing rate assumed in the response spectrum methodology is lower than the one measured on the bridge. A larger spread in the pacing rates, will generate larger response when the modal frequency is away from the average pacing rate.

Podgorica Bridge

The vibration responses calculated for the PB are shown in the second column of Table 2. As in the case of mode 2 on the RCF, first five models tended to overestimate the measured response. The model by Butz (2008a) this time produced a better estimate of the response compared with the estimate for the RCF. Most likely, this is due to more reliable value of k_{red} for cases when actual mean pacing rate in the traffic is closer to the natural frequency of the bridge, as is the case on the PB. The response spectrum also performed better on the PB. On this bridge the problematic assumption (of the pacing rate distribution being narrower than the actual one) still allowed for a number of pedestrians to walk at the pace close to the natural frequency. This prevented underestimation of the measured response, differently from the results on the RCF.

Results from Monte Carlo simulations

In Monte Carlo simulations each person in the traffic flow was represented by an appropriate dynamic force moving along the structure in a prescribed time frame. The individual properties of each pedestrian forcing function were modelled in a probabilistic manner as described in Section 'Monte Carlo simulations'. The distribution of step frequency, which is the most important (i.e. sensitive) parameter in the simulation, was chosen as identified in the measurement tests. The mean weight of pedestrians was taken as 750 N. It was assumed that each person travels along the centreline of the bridge and enters the bridge according to the experimentally identified Poisson distributions.

The result of the MC simulations are time-domain forces for individual pedestrians. After modulating them with the vibration mode of interest and summing them up the total modal force for a mode could be calculated. The modal force is then applied to the SDOF model representing the vibration mode (with modal properties identified experimentally) to calculate the modal vibration response. Using the mode superposition principle (Clough and Penzien, 1993) the total vibration response to pedestrian traffic could be obtained. The total response was composed of contribution of seven modes in RCF having natural frequency below 5 Hz, while it was only one mode in case of the PB.

Traffic simulations were first conducted for the RCF. For the purpose of direct comparison with the benchmark data, each run lasted 16 minutes. However, this (too short) force duration does not provide stationarity (i.e. repetitiveness) of the estimated vibration responses. For this reason the simulations were repeated a number of times and RMS and peak acceleration values were calculated to allow for the comparison with the experimental values equal to 0.06 and 0.36 m.s^2, respectively. Then these quantities were averaged over increasing number of simulations until the convergence (stabilisation) of their averaged value was achieved.

Figure 7a shows the convergence of the averaged total RMS and peak values at TP111 over 20 simulations, as well as the total number of people generated in each simulation (mean value 359 people, coefficient of variation COV=3.6%). It could be seen that the peak acceleration converges to 0.39 m/s^2 (COV=19%) and the RMS value to 0.06 m/s^2 (COV=12%). Their comparison with the measured values reveals that the simulations overestimated the peak value by 8% only, while the RMS value was practically the same as that measured (when two decimal places are accounted for). Therefore, the overall agreement between predicted and measured values is quite good.

The agreement between the simulations and measurements could also be judged in an alternative way. Namely, the time domain signals of all simulated responses and the measured response could be compared in the form of their cumulative probability of nonexceedance of various, say, absolute vibration levels. These are presented for TP111 in Figure 7b separately for the two modes. It can be seen that simulations tended to underestimate the vibration levels in mode 1, which probably is due to experimental overestimate of the modal mass used in simulations, as commented earlier. The agreement in mode 2 is quite good. This suggests that the force model used in simulations is appropriate, especially having in mind that most sensitive parameter in the model, the pacing frequency, is taken from the experimental data.

MC simulations for PB were done in the same manner as for RCF. Each simulation generated 45 minutes of the pedestrian traffic. Various numbers of people were generated in each simulation as shown in Figure 7c, with the mean value of 549 and coefficient of variation (COV) of 3.0%. The stabilisation of averaged peak and RMS responses with increasing number of simulations is shown in the same figure. It can be seen that the peak acceleration converges to 0.99 m/s^2 (COV=14%) and total RMS to 0.22 m/s^2 (COV=8%). Both response estimates are around 65% larger than the corresponding measured values (0.13 m/s^2 for RMS and 0.59 m/s^2 for the peak). The comparison between probability of nonexceedance of absolute vibration levels for simulated (dot-dashed line) and measured (solid line) responses is shown in Figure 7d. For single vibration level the maximum difference is up to 18%, with simulations overestimating the measured response. Having in mind that arrival times and step frequency distribution used in simulations are equivalent to those measured on the bridge, and that the forcing model was successfully applied on the RCF, further analysis of these differences in results for the PB is required.

General comments about performance of the models

The results for Mode 1 are excluded from this discussion due to uncertainty of modal mass parameter used in simulations. It is interesting that the models defined by Sétra (2006) and Brownjohn et al. (2004b) performed quite well on the RCF, being slightly on the conservative side. Also the Monte Carlo simulations produced good estimate of the response. These simulations were done using probabilistic modelling of pedestrians with general parameters (i.e. DLFs, step length, mean pedestrian weight) as defined by Živanović et al. (2007) and with adjusting the arrival times and step frequency distribution to the actual traffic on the bridge. Design procedures that are based on actual traffic modelling (all except EC5 and ISO) account for actual frequency distribution, while arrival time is often embedded in the fitting model and cannot be directly adjusted to the actual traffic. The latter could be a source of discrepancies between the outputs of some models and the measured data, as well as inappropriate calibration of the factor that accounts for pacing frequency distribution.

On the other hand, all models including Monte Carlo simulations, overestimated the response on the PB. Having high confidence in the model behind MC simulations (based on results on more complex RCF structure) as well as confidence in acquired modal properties of the empty PB, there is possibility that these discrepancies are consequence of human-structure interaction (HSI) that is not accounted for in the simulations. It is the authors' experience that when walking across perceptibly moving heavy structures, the pedestrian is influenced by vibration (often felt as feet being hit by the bridge) and forced to walk in an unusual manner. This might result in reduced vibration level with increasing number of people, as in an example demonstrated by Živanović et al. (2009). A way to model this is by increase in damping in the pedestrian-structure system. This is the idea behind simulations explained in the next section.

Improved prediction of vibration response on PB

Monte Carlo simulations

The Podgorica bridge is perceived as lively, and people complain about its vibration (Živanović, 2006). This suggests that some interaction with perceptibly moving structure could be expected, and might be modelled as increased damping of the system. This modelling is attempted here with the aim to improve correlation between calculated and measured responses.

To quantify the additional dampening effect introduced by walking people, a new set of Monte Carlo simulations was conducted assuming various damping ratios associated with the mode 1V. The modal forces generated in the previous 30 simulations were again used to estimate 30 responses (in form of RMS value) with the damping ratio being varied between 0.2% and 1.0%, with a step of 0.05%. Then, in each of 30 simulations, the damping ratio that produces the RMS response closest to the measured value of $0.13\,\mathrm{m/s^2}$ was found. Averaging these damping ratios with increasing number of simulations resulted in an equivalent damping ratio of the human-structure system equal to 0.67% (Figure 8). This is more than two times greater than the damping ratio for the empty bridge (0.26%). The effect of increased damping was not noticed on the RCF due to at least two factors: lower acceleration levels on this bridge and short time of exposure due to short span lengths.

If the newly identified damping ratio of 0.67% is used in 30 simulations to get the response of the Podgorica footbridge then the average estimated vibration measures would converge as follows: peak acceleration to 0.66 m/s^2 (COV=13%) and RMS acceleration to 0.13 m/s^2 (COV=6%), which are much better estimates than the previous ones. The quality of the match between these simulations and measured results could be better seen in Figure 7d. The cumulative distribution of the (absolute) acceleration responses is presented as dashed line. Its agreement with the measurement (solid line) is excellent suggesting that increased damping might be a good way to model the phenomenon.

Performance of models on updated PB

It is interesting to investigate the performance of the models on the PB with updated value of the damping ratio. These results are presented in the third column in Table 2.

EC5 model this time underestimates the response, but, as suggested earlier, have several questionable parameters that make its performance unreliable. ISO 10137, however, still overestimates the peak response. The result suggest that the multiplication factor should be reduced from $N^{0.5}$ to $N^{0.33}$ on this bridge to get best response estimate.

Given that Sétra and UK NA are of similar nature, but still differ in factors that are functions of either mode shape or mean pacing rate in the crowd, the results suggest that performance of UK NA is better in this case. However, the opposite was true for mode 2 on RCF, and therefore it might be that Sétra's ψ factor is more robust than the k factor defined in the UK NA to EC1 model. On the other hand, the factor λ incorporated in the UK NA model seems to improve model performance once k factor is realistic (as in case of the PB). It should be noticed, however, that both codes are referring to modelling much denser traffic than that studied in this paper, and therefore the evaluation of these models on more crowded bridges is required.

The new RMS estimate from Brownjohn's model is very good. This model seems to work well on both bridges, probably due to fact that for the type of traffic investigated little correlation between pedestrians could be expected.

The output of the Butz's model, on the other hand, this time underestimates the measured response, and it it seems that at least its k_{red} parameter needs refinement.

Finally, the estimate from the response spectrum method becomes lower than the measured values. The narrow standard deviation (0.1 Hz) in the distribution of pacing rates used in simulations seems to be main factor responsible for this underestimation.

Discussion and Conclusions

This paper reviews four formal time-domain design procedures used for vibration serviceability assessment of footbridges exposed to multi-person traffic and evaluate their performance in predicting vibration response of two as-built structures. In addition three frequency-domain design procedures, suggested by various authors in last few years are evaluated, as well as a time-domain response estimate obtained from Monte Carlo simulations.

The evaluations suggest that simple procedures, such as EC5 and ISO, are outdated due to oversimplifying the modelling process and treating multi-person traffic as a traffic of smaller number of perfectly correlated people.

The other models (Sétra, UK NA, Butz (2008a) and the response spectrum) are based on extensive numerical simulations that take into account contribution of various structural and traffic parameters to the vibration response. The same applies to the procedure defined by Brownjohn et al. (2004b) that is based on thorough analytical derivations.

A disadvantage of procedure by Butz (2008a) is its limitation to beam-like structures. Other four methods have potential for wider use independent from the structural configuration. However, they all have room for improvement, which is expected in these early stages of their uses. Only further studies of the performance of these models on as-built structures would help their improvement. The time-domain Monte Carlo simulations remain a good means for getting detailed insight into structural response and probability of exceedance of any particular vibration level, as long as the underlaying model for a single pedestrian is appropriate.

However, none of the methods presented accounts for pedestrian-structure interaction when walking across perceptibly oscillating structures. It seems that this interaction in the vertical direction could be modelled as increase in the damping of the system. Therefore there is a strong need for further research on quantification of the HSI, especially on structures which are livelier than the PB studied in this paper. In addition a better statistical characterisation of the vibration response of footbridges is required so that it could be used for verification of existing models for pedestrian traffic. To achieve these advances more full scale measurements on real-life structures are required.

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References

Barker, C., Mackenzie, D. (2008) Calibration of the UK National Annex. In Proceedings of the Third International Conference Footbridge 2008, Porto, 2-4 July.

Brownjohn,J.M.W., Fok,P., Roche,M., Omenzetter,P. (2004a) Long Span Steel Pedestrian Bridge at Singapore Changi Airport - part 2: Crowd Loading Tests and Vibration Mitigation Measures. The Structural Engineer, 82 (16), 28-34.

Brownjohn, J. M. W., Pavić, A., Omenzetter, P. (2004b) A spectral density approach for modelling continuous vertical forces on pedestrian structures due to walking. Canadian Journal of Civil Engineering, 31 (1), 67-77.

BSI (2008) UK National Annex to Eurocode 1: Actions on Structures - Part 2: Traffic Loads on Bridges, NA to BS EN 1991-2:2003, British Standards Institution.

Butz, C. (2006) Beitrag zur Berechnung fußgängerinduzierte Brückenschwingungen. PhD Thesis (in German), ISBN-10 3-8322-5699-7, Shaker Verlag Aachen.

Butz, C. (2008a) A Probabilistic Engineering Load Model for Pedestrian Streams. In Proceedings of the Third International Conference Footbridge 2008, Porto, 2-4 July.

Butz, C. (2008b) Codes of Practice for Lively Footbridges: State-of-the-Art and Required Measures. In Proceedings of the Third International Conference Footbridge 2008, Porto, 2-4 July.

Clough, R. W. and Penzien, J. (1993) Dynamics of Structures. McGraw-Hill, New York.

Dallard, P.; Fitzpatrick, A.J.; Flint, A.; le Bourva, S.; Low, A.; Ridsdill Smith, R. and Willford, M. (2001) The London Millennium Footbridge. The Structural Engineer, 79 (22), 17-33.

EN (2004) Eurocode 5: Design of timber structures – Part 2: Bridges, EN 1995-2:2004, British Standards Institution.

Georgakis, C. T. and Ingólfsson, E. T. (2008) Vertical Footbridge Vibrations: the Response Spectrum Methodology. In Proceedings of the Third International Conference Footbridge 2008, Porto, 2-4 July.

Georgakis, C. T. (2009) Personal communication.

Gudmundsson, G. V., Ingólfsson, E. T., Einarsson, B. and Bessason, B. (2008) Serviceability Assessment of Three Lively Footbridges in Reykjavik. In Proceedings of the Third International Conference Footbridge 2008, Porto, 2-4 July.

HIVOSS (2008) Design of Footbridges Guideline. Human Induced Vibrations of Steel Structures. (http://www.stb.rwth-aachen.de/projekte/2007/HIVOSS/download.php) RFS2-CT-2007-00033.

Ingólfsson, E.T., Georgakis, C., Jönsson, J., Ricciardelli, F. (2007) Vertical footbridge vibrations: towards an improved codifiable response evaluation. The Third International Conference on Structural Engineering, Mechanics and Computation, Cape Town, South Africa, 10-12 September.

Ingólfsson, E. T., Georgakis, C. T. and Svendsen, M. N. (2008) Vertical Footbridge Vibrations: Details Regarding and Experimental Validation of the Response Spectrum Methodology. In Proceedings of the Third International Conference Footbridge 2008, Porto, 2-4 July.

ISO (2007) Bases for design of structures – Serviceability of buildings and walkways against vibration, ISO 10137:2007. International Organization for standardization.

Kasperski, M. (2006) Vibration Serviceability for Pedestrian Bridges. Structures & Buildings, 159 (5), 273-282.

Kerr, S. C. (1998) Human Induced Loading on Staircases, PhD Thesis, University College London, Mechanical Engineering Department, UK.

Macdonald, J. H. G. (2009) Lateral Excitation of Bridges by Balancing Pedestrians. Proceedings of the Royal Society A, 465, 10551073.

Matsumoto, Y., Nishioka, T., Shiojiri, H. and Matsuzaki, K. (1978) Dynamic Design of Footbridges. IABSE Proceedings, No. P-17/78, 1-15.

McRobie, A., Morgenthal, G., Lasenby, J. and Ringer, M. (2003) Section Model Tests on Human-Structure Lock-In. Bridge Engineering, 156 (2), 71-79.

Newland, D. E. (1993) An Introduction to Random Vibrations and Spectral Analysis. Longman Group Ltd, Harlow.

Ricciardelli, F. and Pizzimenti, D. (2007) Lateral Walking-Induced Forces on Footbridges.

ASCE Journal of Bridge Engineering, 12 (6), 677-688.

Roberts, T. M. (2005) Probabilistic Pedestrian Lateral Excitation of Bridges. Proceedings of the Institution of Civil Engineers: Bridge Engineering, 158 (2), 53-61.

Sachse, R. (2002) The Influence of Human Occupants on the Dynamic Properties of Slender Structures, PhD Thesis, University of Sheffield, UK, April.

Sétra (2006) Footbridges, assessment of vibrational behaviour of footbridges under pedestrian loading, Technical guide. Service d'Etudes Techniques des Routes et Autoroutes, Paris.

Venuti, F., Bruno, L. and Napoli, P. (2007) Pedestrian Lateral Action on Lively Footbridges: A New Load Model. Structural Engineering International, 17 (3), 236-241.

Willford, M. (2002) Dynamic actions and reactions of pedestrians. Proceedings of Footbridge 2002, Paris, France, 20-22 November.

Živanović, S. (2006) Probability-Based Estimation of Vibration for Pedestrian Structures due to Walking. PhD thesis, The University of Sheffield, UK.

Živanović, S., Pavić, A. and Reynolds, P. (2006) Modal Testing and FE Model Tuning of a Lively Footbridge Structure. Engineering Structures, 28 (6), 857-868.

Živanović, S., Pavić, A. and Reynolds, P. (2007) Probability Based Prediction of Multi Mode Vibration Response to Walking Excitation. Engineering Structures, 29 (6), 942-954.

Živanović, S., Díaz, I. M. and Pavić, A. (2009) Influence of Walking and Standing Crowds on Structural Dynamic Performance. In Proceedings of IMAC XXVII, Orlando, Florida, USA, 9-12 February.

Živanović, S., Ingólfsson, E. T., Pavić, A. and Gudmundsson, G. V. (2010) Experimental Investigation of Reykjavik City Footbridge. In Proceedings of IMAC-XXVIII, Jacksonville, Florida, 1-4 Feb.

Tables

Table 1: Predicted and measured (in square brackets) acceleration responses [m/s^2] on RCF. Relative percentage error with regard to the measured response is also presented in brackets.

Mode	1	2
EC5	a_{peak}=0.43 [0.12, 258%]	a_{peak}=0.55 [0.31, 77%]
ISO	a_{peak}=0.05 [0.12, -58%]	a_{peak}=1.12 [0.31, 261%]
Sétra	$a_{95\%}$=0.03 [0.11, -73%]	$a_{95\%}$=0.34 [0.27, 26%]
UK NA	$a_{2.5\sigma}$=0.02 [0.08, -75%]	$a_{2.5\sigma}$=0.32 [0.13, 146%]
Brownjohn	a_{RMS}=0.01 [0.03, -67%]	a_{RMS}=0.06 [0.05, 20%]
Butz (2008a)	$a_{95\%}$=0.02 [0.11, -82%]	$a_{95\%}$=0.01 [0.27, -96%]
Response spectrum[1]	$a_{50\%}$=0.03 [0.10, -70%]	$a_{50\%}$=0.09 [0.15, -40%]
Response spectrum[1]	$a_{95\%}$=0.04 [0.11, -64%]	$a_{95\%}$=0.15 [0.27, -44%]

[1] Based on 100 s data blocks.

Table 2: Predicted and measured (in square brackets) acceleration responses [m/s^2] on PB. Relative percentage error with regard to the measured response is also presented in brackets.

Mode	1 ($\zeta = 0.26\%$)	1 ($\zeta = 0.67\%$)
EC5	a_{peak}=1.00 [0.59, 69%]	0.39 [0.59, -34%]
ISO	a_{peak}=1.64 [0.59, 178%]	1.00 [0.59, 69%]
Sétra	$a_{95\%}$=0.70 [0.56, 25%]	0.44 [0.56, -21%]
UK NA	$a_{2.5\sigma}$=0.59 [0.33, 79%]	0.36 [0.33, 9%]
Brownjohn	a_{RMS}=0.20 [0.13, 54%]	0.12 [0.13, -8%]
Butz (2008a)	$a_{95\%}$=0.60 [0.56, 7%]	0.36 [0.56, -36%]
Response spectrum[2]	$a_{50\%}$=0.48 [0.39, 23%]	0.31 [0.39, -21%]
Response spectrum[2]	$a_{95\%}$=0.69 [0.56, 23%]	0.45 [0.56, -20%]

[2] Based on 150 s data blocks.

Figures

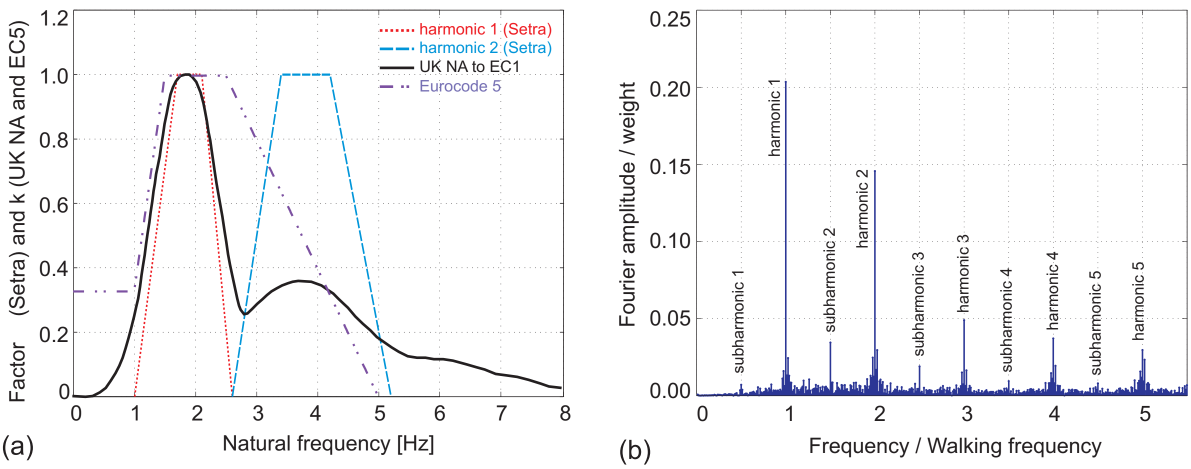

Figure 1: (a) Factors ψ (Sétra) and k (UK NA to EC1 and EC5) as a function of natural frequency and forcing harmonic. (b) Typical frequency content of a walking-induced force measured on an instrumented treadmill.

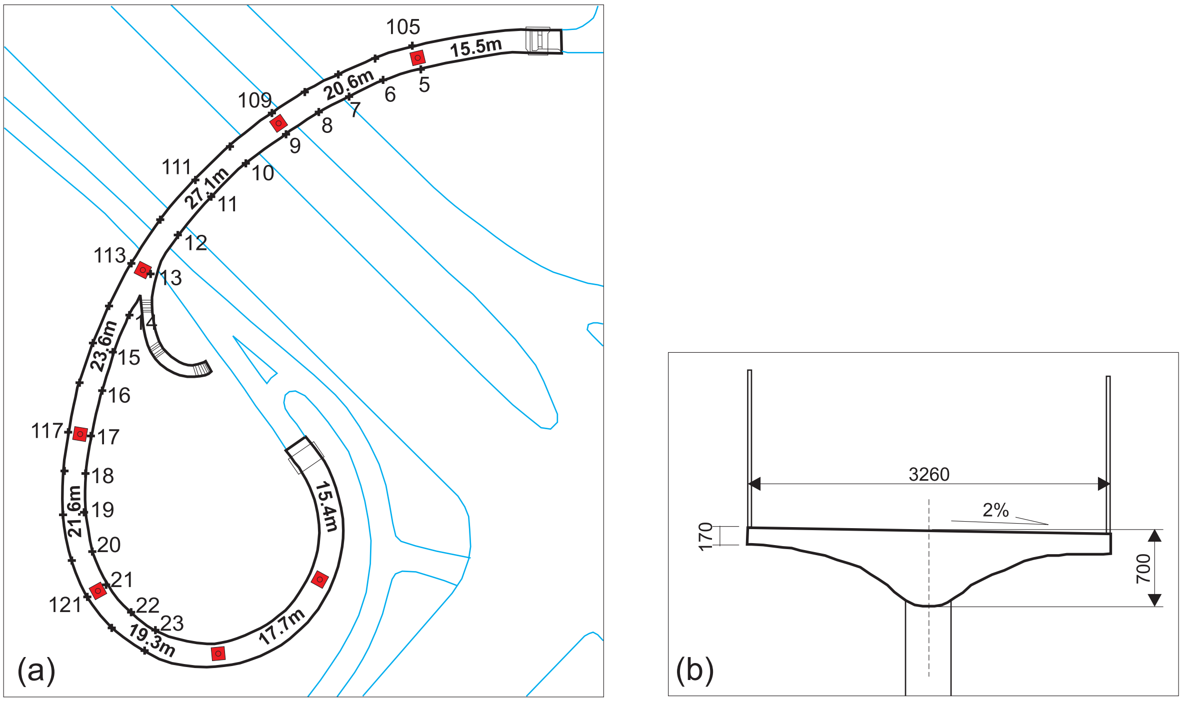

Figure 2: RCF: (a) plan view including test point numbers and (b) cross section (not to scale).

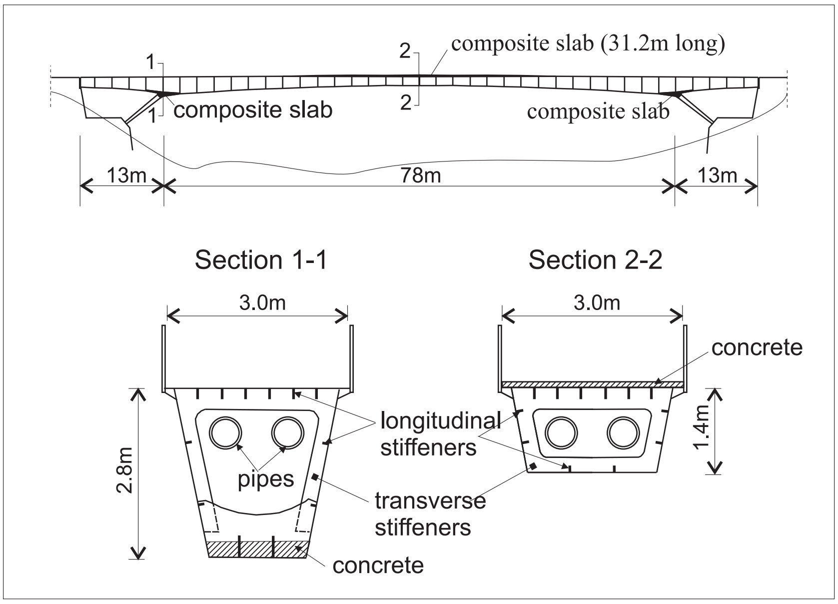

Figure 3: PB: side view and cross sections.

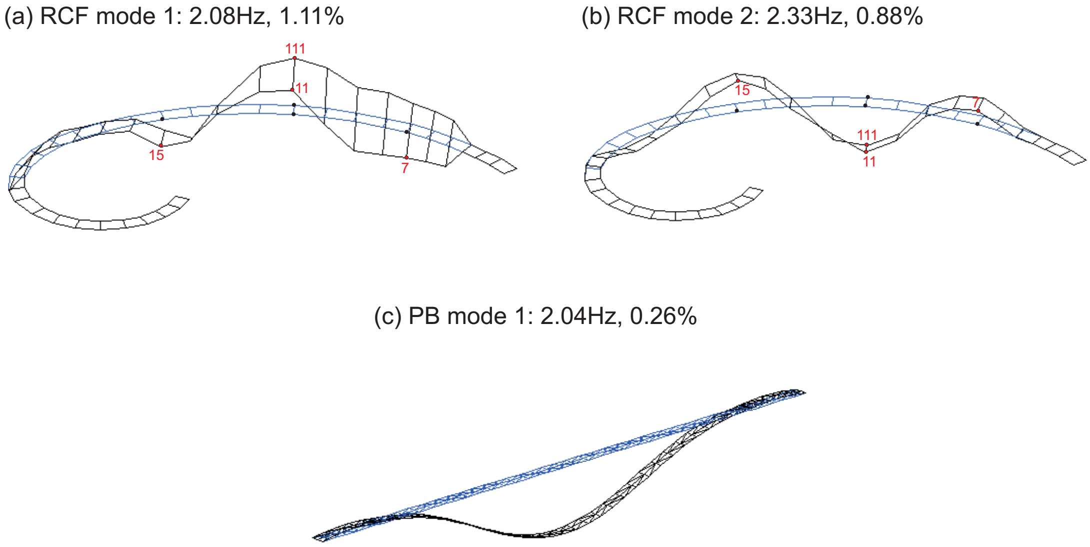

Figure 4: (a and b) Two mode shapes of RCF, and (c) a mode shape for PB, as identified experimentally.

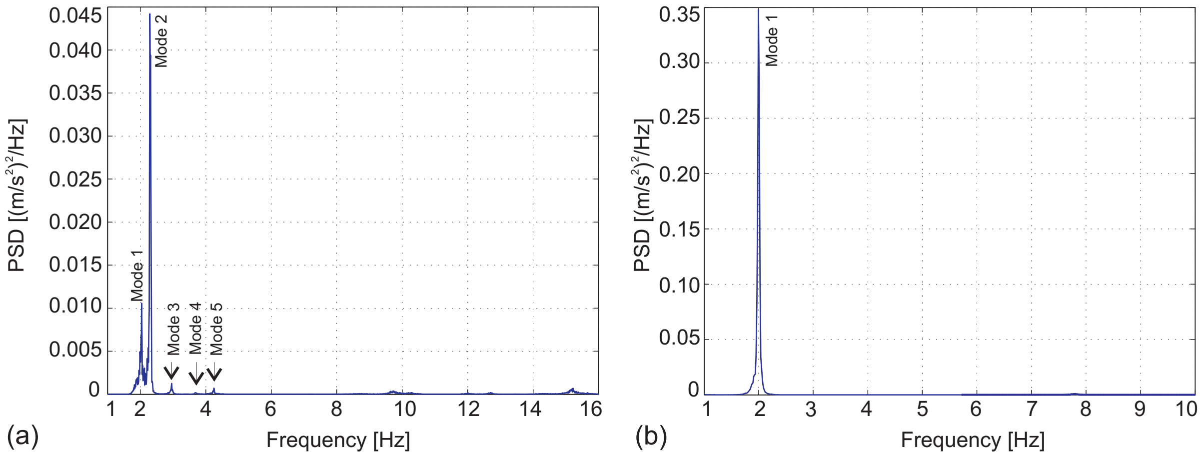

Figure 5: PSD of acceleration measured at (a) TP111 on RCF and (b) the midspan point on PB.

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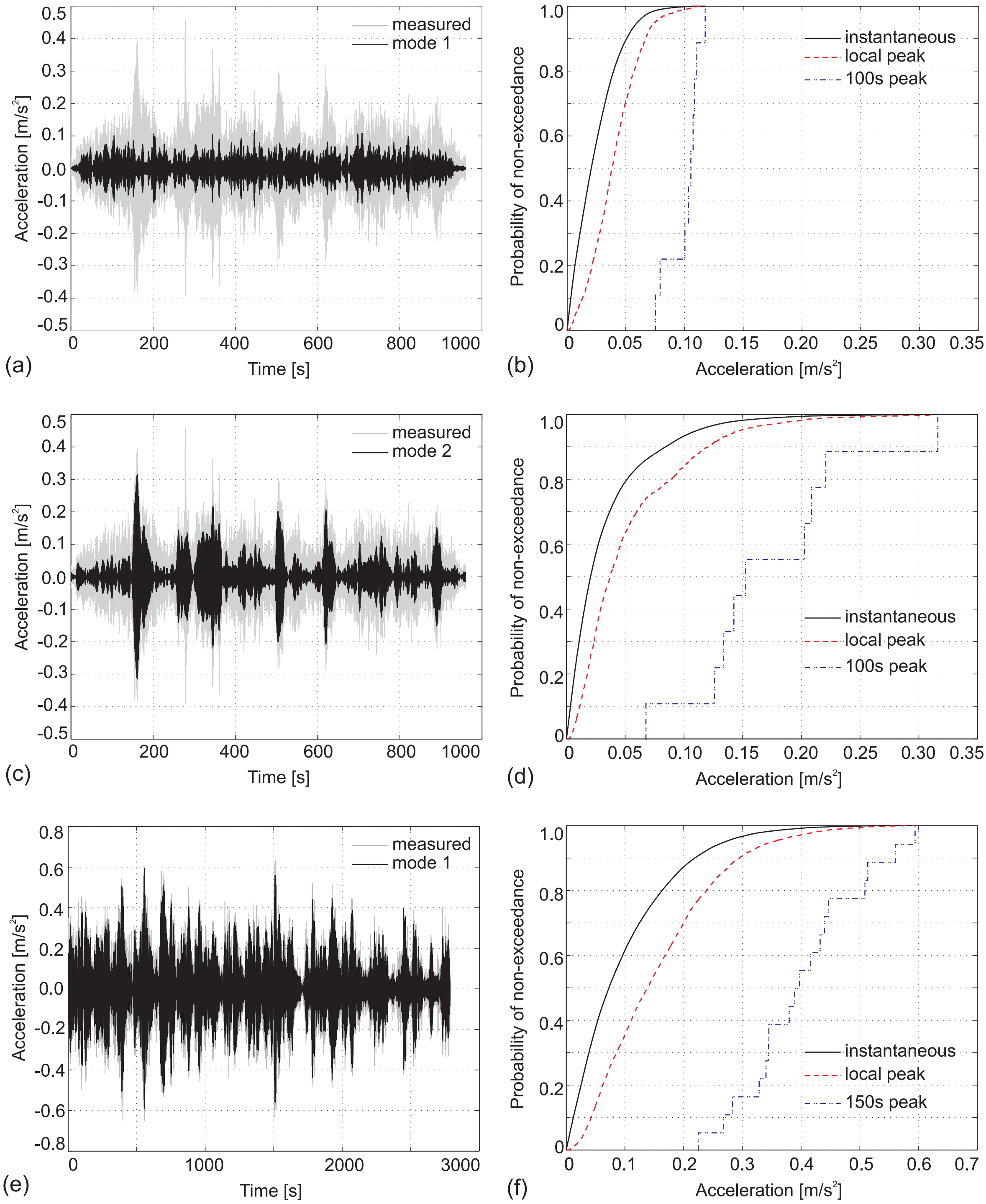

Figure 6: (a) Measured (grey) and modal acceleration in mode 1 on RCF. (b) Cumulative distributions in mode 1 on RCF. (c) Measured (grey) and modal acceleration in mode 2 on RCF. (d) Cumulative distributions in mode 2 on RCF. (e) Measured (grey) and modal acceleration in mode 1 on PB. (f) Cumulative distributions in mode 1 on PB.

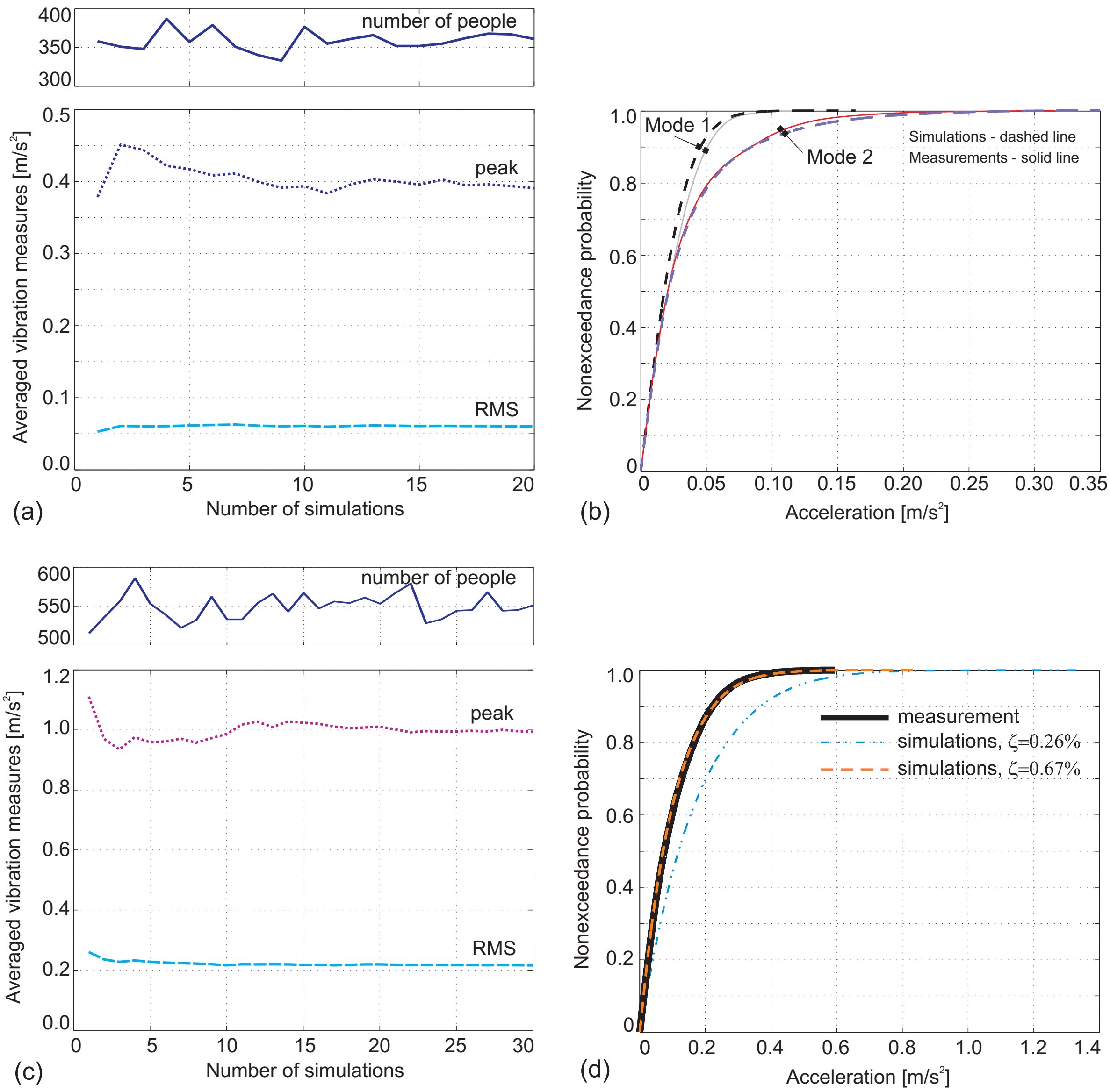

Figure 7: Stabilisation of averaged peak (dotted line) and RMS (dashed line) acceleration with increasing number of simulations on the: (a) RCF and (c) PB, with the total number of people generated in each simulation is shown as solid line. Cumulative distribution of absolute acceleration level at: (b) TP111 on RCF and (d) midspan point on PB.

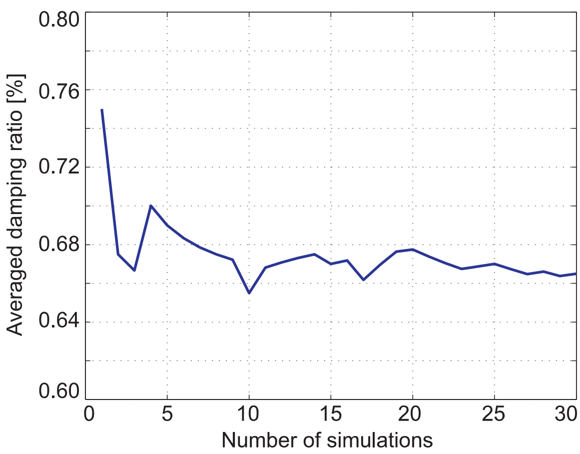

Figure 8: Averaged damping estimate for human-structure system during normal traffic on PB.